Simulation methods in ruin models with non-linear dividend barriers ⋆

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Abstract

In this paper a collective risk reserve process of an insurance portfolio characterized by a homogeneous Poisson claim number process, a constant premium flow and independent and identically distributed claims is considered. In the presence of a non-linear dividend barrier strategy and interest on the free reserve we derive equations for the probability of ruin and the expected present value of dividend payments which give rise to several numerical number-theoretic solution techniques. For various claim size distributions and a parabolic barrier numerical tests and comparisons of these techniques are performed. In particular, the efficiency gain obtained by implementing low-discrepancy sequences instead of pseudorandom sequences is investigated.

Key words: collective risk model, dividend barrier strategies, QMC methods

1 Introduction

Let $\{N(t) : t \in \mathbb{R}_+\}$ denote the random process that counts the claims of an insurance portfolio of a company up to time t and assume that $N(t)$ is a homogeneous Poisson process with intensity λ. Let further $\{X_n : n \in \mathbb{N}\}$ be a sequence of i.i.d. positive random variables with distribution function $F(y)$ representing the sizes of the successive claims and let $\mu = E(X_i) < \infty$. In a time interval $[t, t + dt]$ the company receives the premium $c\,dt$, where $c > \lambda\mu$, and in addition we assume that the company receives interest on its reserves with a constant interest force i (for a general background in ruin theory see e.g. Asmussen [1]). Let T_n $(n \in \mathbb{N})$ denote the moment of occurrence of the nth claim.
We now extend this classical model by introducing a time-dependent dividend barrier b_t, such that whenever the value of the reserve R_t reaches b_t, dividends are paid out to the shareholders with intensity $(c + R_t \cdot i) - db_t$ and the surplus remains on the barrier, until the next claim occurs. This means that the risk process develops according to

$$dR_t = (c + i\,R_t)\,dt - X_{N_t}\,\mathrm{d}N_t \qquad \text{if} \quad R_t < b_t \tag{1}$$

$$dR_t = db_t - X_{N_t}\,\mathrm{d}N_t \qquad \text{if} \quad R_t = b_t. \tag{2}$$

Together with the initial capital $R_0 = u,\ 0 \le u < b_0 < \infty$, this determines the risk process

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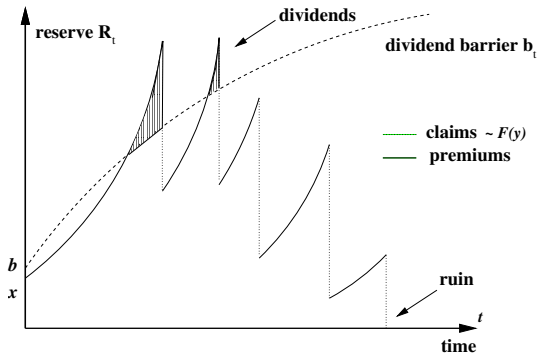

Fig. 1. A sample path of R_t

$\{R_t,\, t \geq 0\}$ (cf. Figure 1). Of particular interest in this context are the survival probability $\phi(u,b) = Pr\{R_t \geq 0 \;\forall\; t \geq 0 \,|\, R_0 = u\}$ and the expected value of the discounted dividend payments $W(u,b)$.

Dividend barrier models have a long history in risk theory [2,3]. In the case of a horizontal dividend barrier $b_t \equiv b_c = \text{const}$, we have $\phi(u,b) = 0 \;\forall\; 0 \leq u \leq b$. Including a constant interest force on the reserve in the model, Paulsen and Gjessing [4] calculated the optimal value of b_c that maximizes the expected value of the discounted dividend payments. For linear dividend barriers $b_t = b + at$ Gerber [5] derived an upper bound for the probability of ruin $\psi(u,b) = 1-\phi(u,b)$ by martingale methods and in [6] he obtained exact solutions for $\psi(u,b)$ and $W(u,b)$ for exponentially distributed claim amounts; this result was generalized by Siegl and Tichy [7] to arbitrary Erlang claim amount distributions, see also Albrecher and Tichy [8]. In [9] non-linear dividend barrier models of the type

$$b_t = \left(b^m + \frac{t}{\alpha}\right)^{1/m} \qquad (\alpha > 0, m \geq 1). \tag{3}$$

were introduced and integro-differential equations for $\phi(u,b)$ and $W(u,b)$ were derived. The existence and uniqueness of the corresponding solutions was discussed and numerical solution techniques were developed and tested for the case of an exponential claim size distribution.

In this paper we extend the results of [9] in various directions in that we add a numerical solution technique based on an iterative scheme, we allow for more general claim size distributions and we consider continuously compounded interest on the free reserve.

In Section 2 we identify $\phi(u,b)$ and $W(u,b)$ as solutions of boundary value problems for integro-differential equations and also as fixed points of contracting integral operators which gives rise to the development of efficient number-theoretic simulation techniques based on Monte Carlo and Quasi-Monte Carlo methods. These are discussed in Section 3. In Section 4 we give numerical results for a parabolic dividend barrier. The various simulation techniques are compared on a quantitative and qualitative basis. Finally the efficiency gain obtained by implementing low-discrepancy sequences is investigated and the sensitivity of the simulation results with respect to the model assumptions is discussed.

2 Integro-differential equations and integral operators

In the sequel we will consider dividend barriers of type (3) (note that $m = 1$ corresponds to the linear barrier case). The probability of survival $\phi(u,b)$ for the surplus process given by (1) and (2) can be expressed as the solution of a boundary value problem. Conditioning

on the occurrence of the first claim, we get for $u < b$

$$\phi(u,b) = (1-\lambda dt)\phi\left(u + (c + i\,u)\,dt, \left(b^m + \frac{dt}{\alpha}\right)^{1/m}\right) + \\ + \lambda\,dt \int_0^{u+(c+i\,u)\,dt} \phi\left(u + (c + i\,u)\,dt - z, \left(b^m + \frac{dt}{\alpha}\right)^{1/m}\right) dF(z). \quad (4)$$

Taylor series expansion of (4) and division by dt shows that ϕ satisfies the equation

$$(c + i\,u)\,\frac{\partial \phi}{\partial u} + \frac{1}{\alpha\,m\,b^{m-1}}\,\frac{\partial \phi}{\partial b} - \lambda\,\phi + \lambda \int_0^u \phi(u-z,b)dF(z) = 0, \quad (5)$$

which, for reasons of continuity, is valid for $0 \le u \le b$. For $u = b$ the same arguments can be used to show that equation (5), with $c + i\,u$ replaced by $\frac{1}{\alpha m\,b^{m-1}}$, also holds. For $b \to \infty$ we additionally require the process to have the survivual probability $\phi(u)$ in the absence of a barrier. Thus we obtain the boundary conditions

$$\frac{\partial \phi}{\partial u}\Big|_{u=b} = 0, \qquad \lim_{b\to\infty} \phi(u,b) = \phi(u). \quad (6)$$

Let furthermore $W(u,b)$ denote the expected present value of the future dividend payments, which are discounted according to the riskless interest rate i, and stop when ruin occurs. Then, in a similar way to (4), one can derive the integro-differential equation

$$(c + i\,u)\,\frac{\partial W}{\partial u} + \frac{1}{\alpha\,m\,b^{m-1}}\,\frac{\partial W}{\partial b} - (i+\lambda)\,W + \lambda \int_0^u W(u-z,b)dF(z) = 0, \quad (7)$$

with boundary condition $\frac{\partial W}{\partial u}\Big|_{u=b} = 1$.

Remark 1: In principle, one could follow this approach for any dividend barrier function $b_t = f(b,t)$ that is monotone increasing in t and satisfies

$$f(b,t) = f\Big(f(b,t_1), t - t_1\Big) \quad \forall\, b > 0 \text{ and } \forall\, t > t_1 > 0. \quad (8)$$

The functional equation (8) is the well-known translation equation and for functions $f(b,t)$ which are monotone increasing in b and t and continuous in b, the general solution of (8) is given by

$$f(b,t) = h\Big(h^{-1}(b) + t\Big),$$

where $h(t) = f(b_0,t)$ is some given initial function (see e.g. Aczél [10]). From $h(t) = (b_0^m + t/\alpha)^{\frac{1}{m}}$ we obtain (3) as a special case. Other solutions of (8) include for instance $f(b,t) = b + at$ (linear barrier) or $f(b,t) = (\sqrt{b} + t)^2$ (quadratic barrier).

Remark 2: For the special case of exponentially distributed claim sizes it follows from (5) and (7) that $\phi(u,b)$ and $W(u,b)$ can be expressed as the solutions of boundary value problems for second-order partial differential equations of hyperbolic type. However, due to the structure of the boundary conditions this does not lead to a simplification of the problem (cf. [9]).

Generalizing a procedure developed in [6] for the case of linear barriers, we first show

that the boundary value problem (7) has a unique bounded solution. For that purpose, we define an operator A by

$$Ag(u,b) = \int_0^{t^*} \lambda e^{-(\lambda+i)t} \int_0^{(c'+u)e^{it}-c'} g\left((c'+u)e^{it}-c'-z, \left(b^m+\frac{t}{\alpha}\right)^{1/m}\right) dF(z)dt+$$
$$+\int_{t^*}^{\infty} \lambda e^{-(\lambda+i)t} \int_0^{\left(b^m+\frac{t}{\alpha}\right)^{1/m}} g\left(\left(b^m+\frac{t}{\alpha}\right)^{1/m}-z, \left(b^m+\frac{t}{\alpha}\right)^{1/m}\right) dF(z)dt+$$
$$+\int_{t^*}^{\infty} \lambda e^{-\lambda t} \int_{t^*}^{t} e^{-is}\left((c+i\,u)e^{is}-\frac{1}{m\alpha\left(b^m+\frac{s}{\alpha}\right)^{1-1/m}}\right) ds\; dt, \quad (9)$$

with $c' = c/i$. Here t^* is the positive solution of $(c'+u)e^{it}-c' = \left(b^m+\frac{t}{\alpha}\right)^{1/m}$ (this number is unique for all $m \geq 1$, since $u < b$). The solution $W(u,b)$ of (7) with its initial condition is a fixed point of the integral operator A. For any two bounded functions g_1, g_2

$$|Ag_1(u,b) - Ag_2(u,b)| \leq \|g_1 - g_2\| \int_0^{\infty} \lambda e^{-(\lambda+i)t} dt \leq \frac{\lambda}{\lambda+i} \|g_1 - g_2\| \quad (10)$$

for arbitrary $0 \leq u \leq b < \infty$, where $\|\cdot\|$ is the supremum norm on $0 \leq u \leq b < \infty$, and thus it follows that A is a contraction in the Banach space of bounded functions equipped with the supremum norm, and the fixed point is unique by Banach's theorem.
Unfortunately, the same approach does not allow to show the contraction property of the corresponding integral operator for the probability of survival $\phi(u,b)$. However, if we introduce a horizontal absorbing upper barrier $b_{max} = const$ (i.e. if the surplus process $R_t \geq b_{max}$ for some $t > 0$, it is absorbed, the dividend payments stop and the company is considered to have survived), then one can derive contracting integral operators similar to (9) for $W(u,b)$ and $\phi(u,b)$. From an economic point of view, this slight modification of the risk model (called Model B in the sequel) can be interpreted that the company will then decide to pursue other forms of investment strategies.
Since these boundary value problems can not be solved analytically, there is a need for effective algorithms to obtain numerical solutions. In this paper we focus on the development of number-theoretic solution methods based on the corresponding integral operators and on stochastic simulation, respectively.

3 Numerical solution techniques

The following three algorithms will be presented in terms of operator (9).

3.1 Double-recursive Algorithm

The fixed point of (9) can be approximated by applying the contracting integral operator A k times to a starting function $g^{(0)}(u,b) := h(u,b)$ which we choose to be the inhomogeneous term in the corresponding integral operator (where k is chosen according to the desired accuracy of the solution):

$$g^{(k)}(u,b) = A^k g^{(0)}(u,b),$$
$$g^{(0)}(u,b) = h(u,b) := \int_{t^*}^{\infty} \lambda e^{-\lambda t} \int_{t^*}^{t} e^{-is}\left((c+i\,u)e^{is}-\frac{1}{m\alpha\left(b^m+\frac{s}{\alpha}\right)^{1-1/m}}\right) ds\; dt.$$

This leads to a $2k$-dimensional integral for $g^{(k)}(u,b)$, which is calculated numerically using Monte Carlo and Quasi-Monte Carlo methods. The Monte Carlo-estimator of $W(u,b)$ for given values of u and b then is

$$W(u,b) \approx \frac{1}{N}\sum_{n=1}^{N} g_n^{(k)}(u,b)\ , \tag{11}$$

where the $g_n^{(k)}(u,b)$ are calculated recursively for each n by

$$g_n^{(i)}(u,b) = h(u,b) + \frac{\lambda}{\lambda+i}\cdot$$
$$\cdot\Bigg\{F\left((c'+u)e^{it_{1,n}^i}-c'\right)\left(1-e^{-(\lambda+i)t^*}\right)g_n^{(i-1)}\left((c'+u)e^{it_{1,n}^i}-c'-z_{1,n}^i,\left(b^m+\frac{t_{1,n}^i}{\alpha}\right)^{\frac{1}{m}}\right)+$$
$$+F\left(\left(b^m+\frac{t_{2,n}^i}{\alpha}\right)^{\frac{1}{m}}\right)e^{-(\lambda+i)t^*}g_n^{(i-1)}\left(\left(b^m+\frac{t_{2,n}^i}{\alpha}\right)^{\frac{1}{m}}-z_{2,n}^i,\left(b^m+\frac{t_{2,n}^i}{\alpha}\right)^{\frac{1}{m}}\right)\Bigg\}.$$

$(1\le i\le k)$ and $g_n^{(0)}(u,b)=h(u,b)$. Here $t_{j,n}^i$ and $z_{j,n}^i$ $(j=1,2)$ are determined according to

$$t_1 = -\frac{\log\left(1-w_1\left(1-e^{-(\lambda+i)t^*}\right)\right)}{\lambda+i} \qquad z_1 = F^{-1}\left(v_1\cdot F\left((c'+u)e^{it}-c'\right)\right) \tag{12}$$

$$t_2 = t^* - \frac{\log(1-w_2)}{(\lambda+i)} \qquad z_2 = F^{-1}\left(v_2\cdot F\left(\left(b^m+\frac{t_2}{\alpha}\right)^{\frac{1}{m}}\right)\right). \tag{13}$$

for (quasi-)random deviates v_j, w_j of the uniform distribution in the unit interval.
Since in every recursion step the function g is called twice, the number of evaluations of g doubles in every recursion step. Thus, in order to keep the computations tractable, in what we will call the double-recursive algorithm in the sequel, the double recursion is only used for the first three recursive steps and for the remaining recursion steps the recursive algorithm described in Section 3.2 is applied.

3.2 Recursive Algorithm

Instead of calculating the first two integrals occurring in operator (9) separately, one can combine them to one integral. A suitable change of variables then leads to

$$Ag(u,b) = h(u,b)+$$
$$\int_0^1\int_0^1 \frac{\lambda}{\lambda+i}F\left(z_{max}(u,b,t)\right)g\left(z_{max}(u,b,t)-z,\left(b^m+\frac{t}{\alpha}\right)^{\frac{1}{m}}\right)dvdw \tag{14}$$

where t and z are given by

$$t = -\frac{\log(1-w)}{(\lambda+i)}, \qquad z = -F^{-1}\left(v\cdot F(z_{max}(u,b,t))\right) \tag{15}$$

and $z_{max}(u,b,t)$ is defined as the minimum of the reserve and the dividend barrier at time t. Note that for this algorithm, the number of integration points needed for a given recursion depth is one eighth of the corresponding number required for the double-recursive case.

3.3 Iterative Algorithm

Another solution technique based on the integral operator (14) is to discretize the domain of u and b by a grid (u_j, b_k), $0 \le j \le j_{max}$, $0 \le k \le k_{max}$. After assigning a suitable initial value to each discretization point (u_j, b_k), the operator is applied sequentially to each point (u_j, b_k) of the grid. The resulting approximative solution $\hat{g}^{(i)}_{j,k}$ at point (u_j, b_k) and iteration depth i is calculated from the values of $\hat{g}^{(i-1)}$ at depth $(i-1)$ by a two-dimensional integral, which is evaluated by Monte Carlo and Quasi-Monte Carlo methods. Since $\hat{g}^{(i-1)}$ is only defined for the discretization points (u_j, b_k), the function g in operator (14) is replaced by a linear interpolation function I. Thus we have

$$\begin{aligned}
\hat{g}^{(0)}_{j,k} =& h(u_j, b_k) \\
\hat{g}^{(i)}_{j,k} =& h(u_j, b_k) + \frac{\lambda}{\lambda + i} \cdot \\
& \frac{1}{N} \sum_{n=1}^{N} F\left(z_{max}(u_j, b_k, t^i_n)\right) I\left(\hat{g}^{(i-1)}, z_{max}(u_j, b_k, t^i_n) - z^i_n, \left(b^m_k + \frac{t^i_n}{\alpha}\right)^{\frac{1}{m}}\right)
\end{aligned} \tag{16}$$

for $1 \le i \le i_{max}$. The variables (t^i_n, z^i_n) are obtained from the elements of a 2-dimensional sequence according to (15), and the iteration depth i_{max} is chosen according to the desired accuracy of the approximation.

3.4 Simulation

Another way to obtain numerical solutions is stochastic simulation of the surplus process. For that purpose we sample N paths of the risk reserve process and use the Monte Carlo estimates $\phi(u,b) \approx \frac{m}{N}$ and $W(u,b) \approx \frac{1}{N}\sum_{k=1}^{N} v(k)$, where m is the number of simulated paths for which ruin does not occur and $v(k)$ is the sum of discounted dividend payments of path k. We consider a simulation path as having survived, if it exceeds a given threshold x_{max}, and so the process stops with probability 1. Using this stopping criterion, we overestimate (underestimate, resp.) the actual probability of survival $\phi(u,b)$ ($W(u,b)$, resp.). For sufficiently large x_{max}, however, this effect is negligible.

3.5 Quasi-Monte Carlo Approach

All the numerical solution techniques described above lead to the numerical evaluation of integrals, which in the classical Monte Carlo algorithm is done by using pseudo-random numbers. However, the use of deterministic uniformly distributed instead of pseudo-random point sequences has proven to be an efficient extension of the classical Monte Carlo method. A well-known measure for the uniformness of the distribution of a sequence $\{\mathbf{x}_n\}_{1 \le n \le N}$ in $U^s := [0,1)^s$ is the star-discrepancy

$$D^*_N(\mathbf{x}_n) = \sup_{I \in J^s_0} \left| \frac{A(\mathbf{x}_n; I)}{N} - \lambda_s(I) \right| ,$$

where J^s_0 is the set of all intervals of the form $[0, \vec{y}) = [0, y_1) \times [0, y_2) \times \ldots \times [0, y_s)$ with $0 \le y_i < 1$, $i = 1, \ldots, s$ and $A(\mathbf{x}_n; I)$ is the number of points of the sequence $\{\mathbf{x}_n\}_{1 \le n \le N}$ that lie in I. $\lambda_s(I)$ denotes the s-dimensional Lebesgue-measure of I.

The notion of discrepancy is particularly useful for obtaining an upper bound for the error of Quasi-Monte Carlo integration, since by the Koksma-Hlawka inequality, we have

$$\left| \frac{1}{N} \sum_{n=1}^{N} f(x_n) - \int_{[0,1)^s} f(u)du \right| \leq V(f) D_N^* (x_1, \ldots, x_N) \ . \tag{17}$$

for any $\{x_1, \ldots, x_N\} \subset [0,1)^s$ and for any function $f : [0,1)^s \to \mathbb{R}$ of bounded variation $V(f)$ in the sense of Hardy and Krause (see e.g. [11]). In contrast to error bounds in classical Monte Carlo, (17) is a deterministic error bound. Especially for s not too large, low discrepancy sequences, i.e. sequences for which

$$D_N^* (x_1, \ldots, x_N) \ \leq C_s \frac{(\log N)^s}{N}, \tag{18}$$

with an explicitly computable constant C_s holds, have turned out to be superior to pseudo-Monte Carlo sequences in many applications. Typical examples are the Halton sequence [12] and sequences based on (t, m, s)-nets such as the Sobol, Faure and Niederreiter-(t, s) sequences [13–15].

4 Numerical Results for the parabolic case

In this section we present numerical results for a parabolic dividend barrier of the form $b_t = \sqrt{b^2 + t/\alpha}$ and various claim amount distributions (with coinciding mean μ and variance σ^2). The domain of u and b in the following simulations is a grid in the triangle $(b = 0..[0.1]..1, u = 0..[0.1]..b)$, and the parameters are chosen in the following way: $c = 1.5$, $i = 0.1$, $\alpha = 0.5$, $\lambda = \gamma = 1$, $\mu = 1$, $\sigma^2 = 0.5$, $i_{max} = 66$, $b_{\max} = 4$. The recursion depth for the recursive methods is $k = 66$. Moreover $N = 66\,000$ (recursive method, simulation) and $N = 33\,000$ (iterative, double recursive method). Since exact values are not available, we use a MC-estimator over 10 million paths for every choice of u and b.
All our QMC-calculations are actually hybrid Monte Carlo estimates, i.e. the initial 50 dimensions are generated by a 50-dimensional QMC sequence and the remaining dimensions are generated by a pseudo-random number generator.The use of hybrid Monte Carlo sequences has proven to be a successful modification of the QMC-technique, since for low discrepancy sequences typically the number of points needed to obtain a satisfying degree of uniformness dramatically increases with the dimension. Moreover, due to the nature of our risk process, the initial dimensions of the sequence have a higher impact on the solution than higher dimensions. Throughout this paper, we use `ran2` as our pseudo-random number generator, which basically is an improved version of a minimal standard generator based on a multiplicative congruential algorithm [16].
The different methods and sequences used are compared via the mean square error (MSE)

$$S = \sqrt{\frac{1}{|P|} \sum_{(u,b) \in P} \Big(g(u,b) - \tilde{g}(u,b) \Big)^2}, \tag{19}$$

where $g(u,b)$ and $\tilde{g}(u,b)$ denote the exact and the approximated value, respectively, and the set P is the grid in the triangular region $0 \leq u \leq b \leq 1$ described above. To quantify the effect of using a low discrepancy sequence, we perform a regression analysis by fitting

$$\log_2(S) = a_0 + a_1 \log_2(N) + a_2 \log_2(\log_2(N)) + \epsilon \tag{20}$$

to the data using a least square fit. Note that Koksma-Hlawka's inequality (17) could be interpreted as implying $a_1 = -1$ and $a_2 = s$, where s is the dimension of the sequence used. However, since we use a hybrid sequence and since the effective dimension is smaller than the theoretical dimension, the values of a_1 and a_2 deviate from the ones above.

4.1 Error analysis of the survival probability

The computations show that for Model B (introduced in Section 2), the convergence in terms of N of the approximations to the exact solution is much better for the Halton and Sobol sequence then for MC-estimators (see Fig. 2 for Pareto claim sizes; other distributions show a similar behavior). In absence of an absorbing upper barrier b_{max}, this effect does not occur (the effective dimension of the latter problem is much larger).

4.2 Error analysis of the expected value of the dividend payments

The simulation results for the expected value of the dividends show a clear advantage of QMC methods over MC integration (both for Model A and B). As an illustration, Fig. 3 depicts the MSE of the simulation results as a function of N for the Weibull distribution. While for small N, the Sobol sequence clearly outperforms the other sequences in terms of the MSE for given N, the Halton sequence used in the recursive algorithm shows a nice convergence rate for large N. This effect can be quantified by introducing the efficiency gain $N^*_{\mathrm{MC}}(S)/N^*_i(S)$, where $N^*_{\mathrm{MC}}(S)$ is the number of paths needed in the Monte Carlo simulation to reach a given error of S, and $N^*_i(S)$ is the corresponding N using an alternative method. Fig. 4 shows the efficiency gains as a function of N.
If one considers the approximation error as a function of calculation time, the double recursive algorithm is still competitive (see Fig. 5); however, the recursive method using Sobol's sequence seems preferable. Furthermore, Fig. 5 allows for a comparison of the iterative method with recursive and simulation methods in terms of efficiency.
Since the effective dimension is much smaller in Model B, one can expect low-discrepancy sequences to outperform pseudo-random sequences to an even larger extent. Simulations show that this is indeed the case and for small N the Sobol sequence shows the best performance, whereas for larger N Niederreiter-(t,s)-sequences are preferable (on the other hand, Niederreiter-$(0,s)$- and Faure sequences turn out not to be suited for the integrands of our problems - their performance is rather poor). The performance of Niederreiter-(t,s)-sequences can still be improved by starting the sequence at $n = p^k$ with k being at least the maximum degree of the polynomials used to generate the sequence and p denoting the base of the construction (this effect is known as the leading-zeros phenomenon, see also [17,18]). Table 1 shows the approximation errors of the various solution methods for a Weibull claim size distribution. A complete list of exact values and simulation results can be obtained from the authors.

		Monte Carlo	Halton	Niederr. (t,s)	Sobol
Simulation	S	0.00533089	0.000849476	0.000803501	0.000875673
	$\|\Delta\|_\infty$	0.0136	0.00242	0.00216	0.00203
Recursive	S	0.00257658	0.000570681	0.000486129	0.000688766
	$\|\Delta\|_\infty$	0.00652	0.00178	0.00158	0.00207
Double Rec.	S	0.00241979	0.000619308	0.000643352	0.000921779
	$\|\Delta\|_\infty$	0.00572	0.00191	0.00196	0.00243
Iterative	S	0.016305	0.0170657	0.0169917	0.016983
	$\|\Delta\|_\infty$	0.02382	0.02149	0.02138	0.02138

Table 1: MSE and maximum error for $W(u,b)$ (Weibull distribution, Model B)

4.3 Model Analysis

In order to investigate the sensitivity of $\phi(u,b)$ and $W(u,b)$ to the claim size distribution and to the consideration of interest rates in the model, we fix a value of b and plot $\phi(u,b)$ and $W(u,b)$ against u for $i = 0.1$ and $i = 0$, resp. (see Fig. 6 and 7). Fig. 8 shows the survival probability for a larger range of u (where the heavy-tail property of the claim size distribution becomes relevant). Our simulations show that, in contrast to $W(u,b)$, for $\phi(u,b)$ the choice of the claim size distribution in the model is important. Both $\phi(u,b)$ and $W(u,b)$ are sensitive to the inclusion of interest on the free reserve in the model.

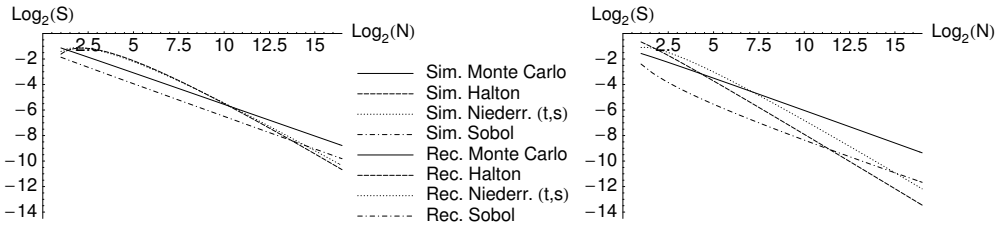

Fig. 2. MSE of $\phi(u,b)$ estimates as a function of N (Pareto distribution, Model B)

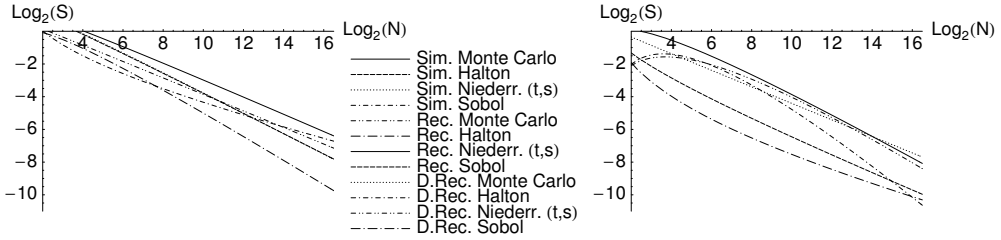

Fig. 3. MSE of $W(u,b)$ (Weibull distribution, Model A)

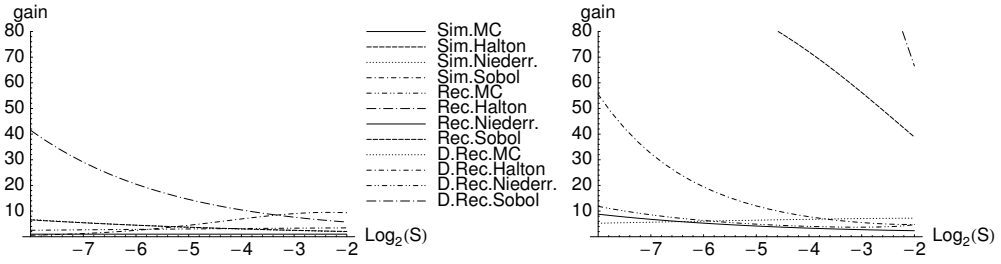

Fig. 4. Efficiency gain of the solution methods of Figure 3

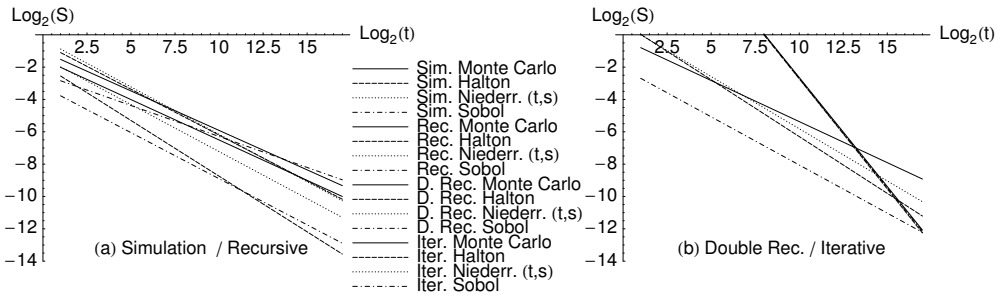

Fig. 5. MSE of $W(u,b)$ vs. calculation times in seconds (Pareto distribution, Model A)

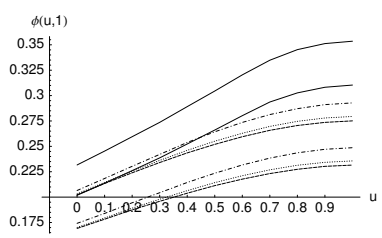

Fig. 6. $\phi(u,1)$ in Model A

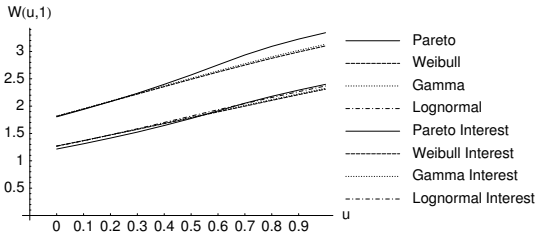

Fig. 7. $W(u,1)$ in Model B

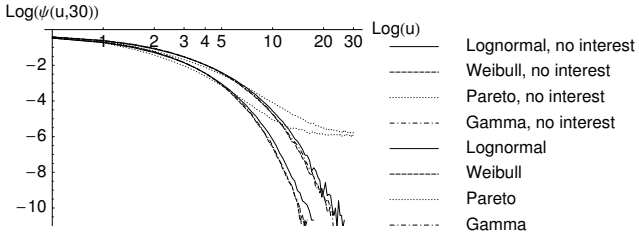

Fig. 8. Log-log plot of $\psi(u,30) = 1 - \phi(u,b)$ in Model A

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