QMC integration of improper integrals[1]

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In financial applications of QMC methods, often integrals appear with a singular integrand on the integration boundary. As the variation in the sense of Hardy and Krause is not bounded for such functions, one has to use other theorems than the Koksma-Hlawka inequality to show convergence of QMC methods and estimate the convergence rate. Recently, a lot of research effort has been spent on this case, beginning with Sobol' [1], Klinger [2], and de Doncker and Guan [3].
In 2003, Hartinger, Kainhofer, and Tichy [4] investigated the problem of singular integrands and non-uniform integration measures, which was later applied to the problem of evaluating Asian options by Hartinger, Kainhofer, and Predota [5]. It turns out that in this unbounded case (which fulfills the prerequisites of Hartinger, Kainhofer, and Tichy's paper) the order of the integration error is almost as good as the QMC error for "well-behaving" integrands of bounded variation.
Lately, Owen [6] even proved that the Halton sequence avoids certain hyperbolic regions around the corners of the integration interval, and thus is particularly well suited for singular integrands.

In this talk I will give a quick overview of the integration of singular integrands and present some ideas that show how one can safely use QMC integration for singular integrands without loosing the good error order of QMC methods. Both uniformly and non-uniformly distributed sequences will be addressed and error orders will be proved.

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